POLITECNICO DI MILANO

Deterministic asynchronous two-machine lines

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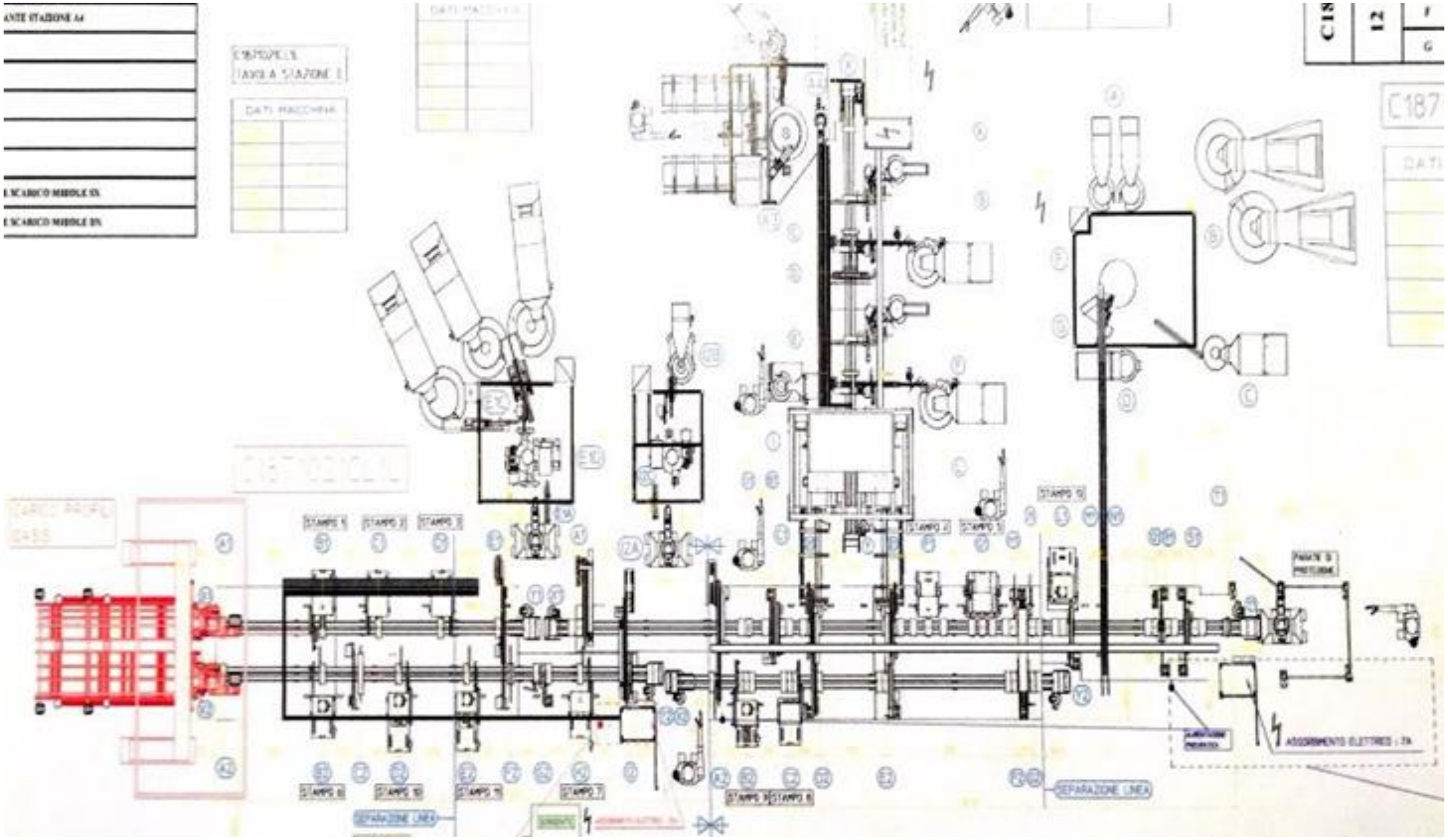
Outline

1. Motivation
2. Single up – single down continuous model
3. Proposed approach
4. Numerical results
5. Conclusions and future research

1. Motivation

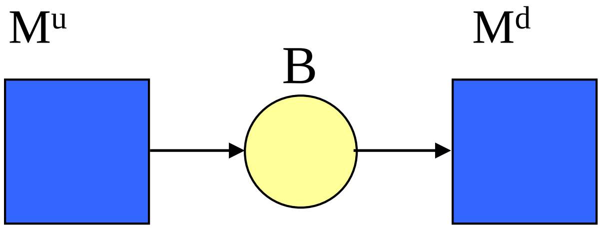
- **Automated production line**



(Real system from Cosberg)

1. Motivation

➢ **Reference system**



$(\mu^u > \mu^d)$

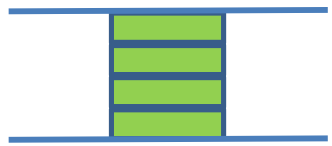
- parts are discrete and each machine processes one part at a time;
- the system is asynchronous i.e. each machine can start or finish a part at any time without synchronization with the other machine;
- processing times are deterministic and may be different among the machines;
- the blocking discipline is Blocking After Service (BAS).

1. Motivation

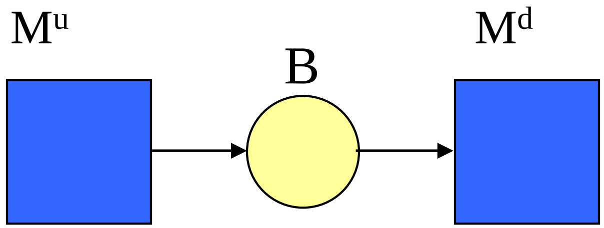
How can we model it ?

- Discrete model
- Exponential model
- Continuos model

- **Reference system**



$(\mu^u > \mu^d)$



1. Motivation

In continuous time models the discrete nature of the physical parts is lost. This may result in approximations when the buffer level is close to be empty or full.

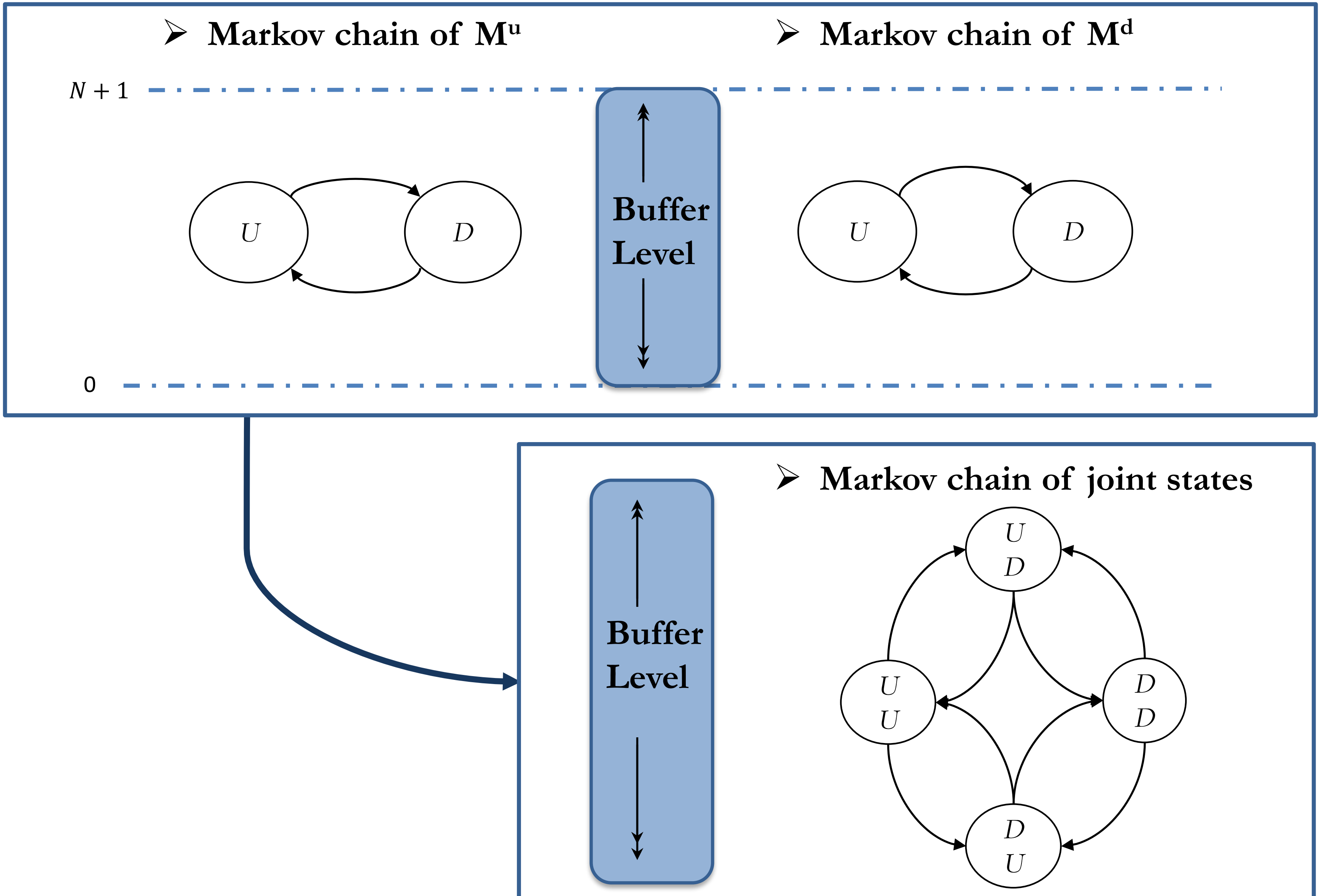
The **intent** of the present work is to propose an exact continuous model to approximate the behavior of deterministic asynchronous two-machine lines with finite buffer capacity, which works well even when the buffer is close to be empty or full.

2. Single up – single down continuous model

Assumptions

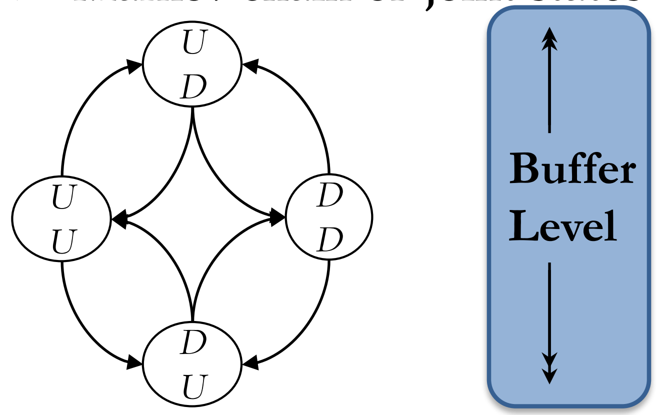
- each machine can be in one of the two states: up or down.
- Time To Failure and Time To Repair have exponential distributions.
- machine failures happen at the beginning of the operations on a part. Therefore when a failure happens there are no parts partially machined on the machine.

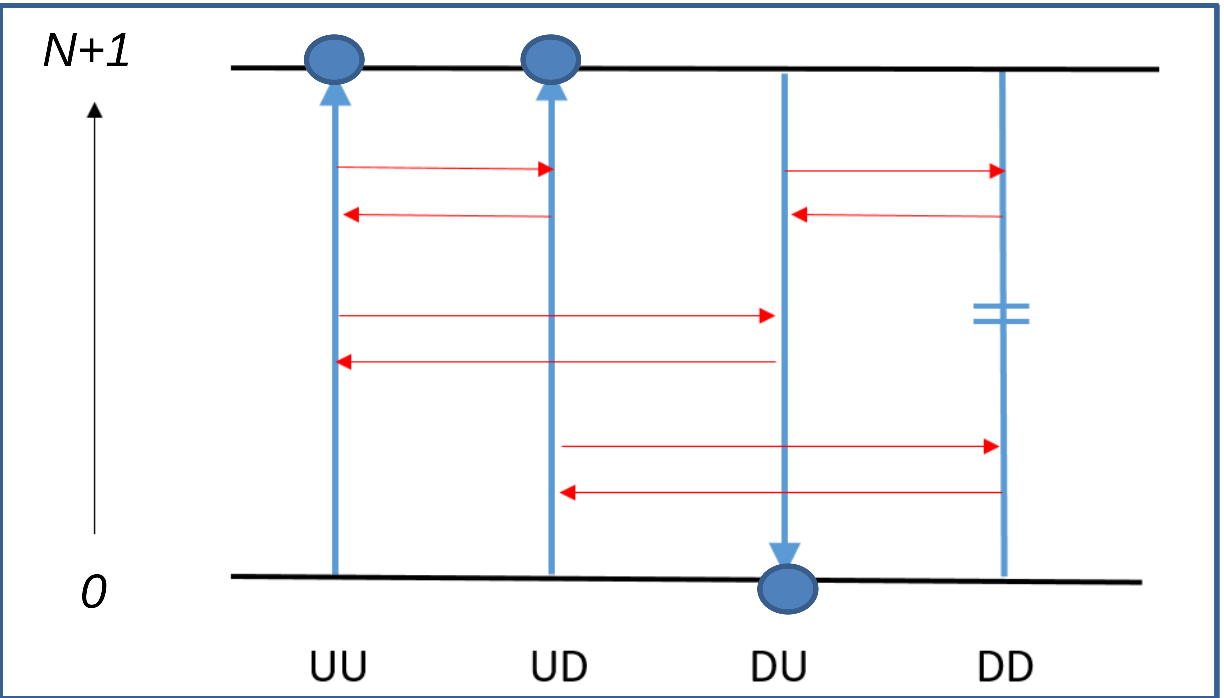
2. Single up – single down continuous model



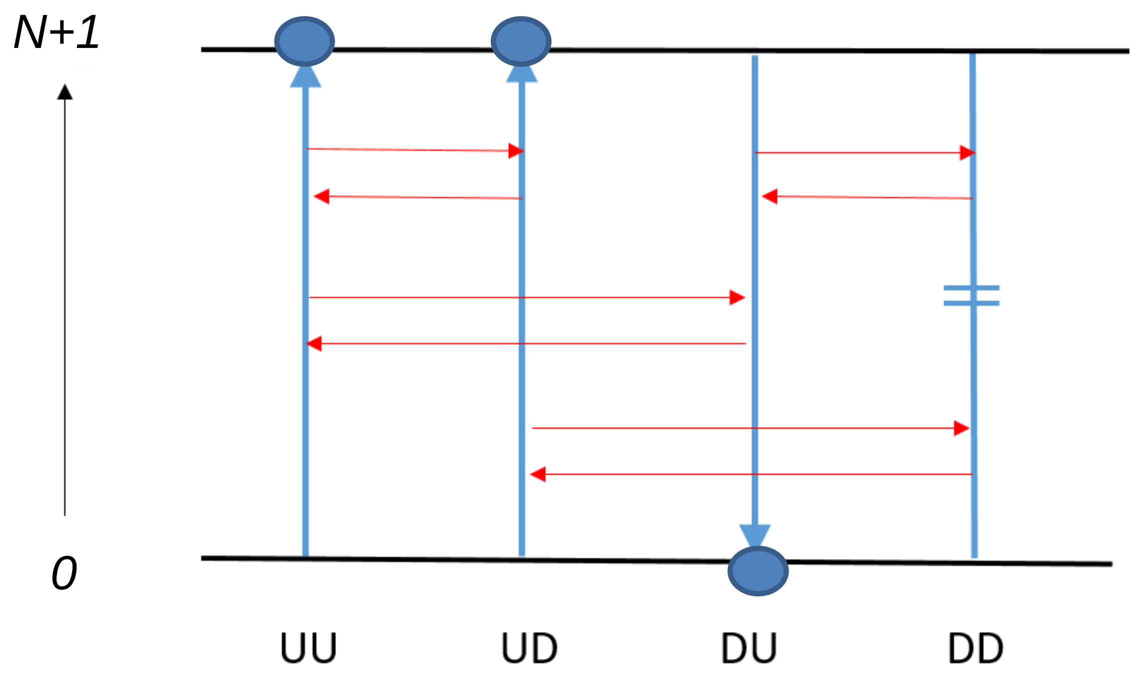
2. Single up – single down continuous model

- Markov chain of joint states



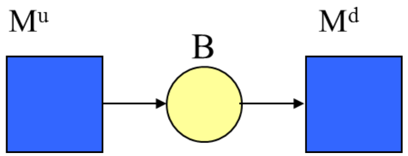


2. Single up – single down continuous model

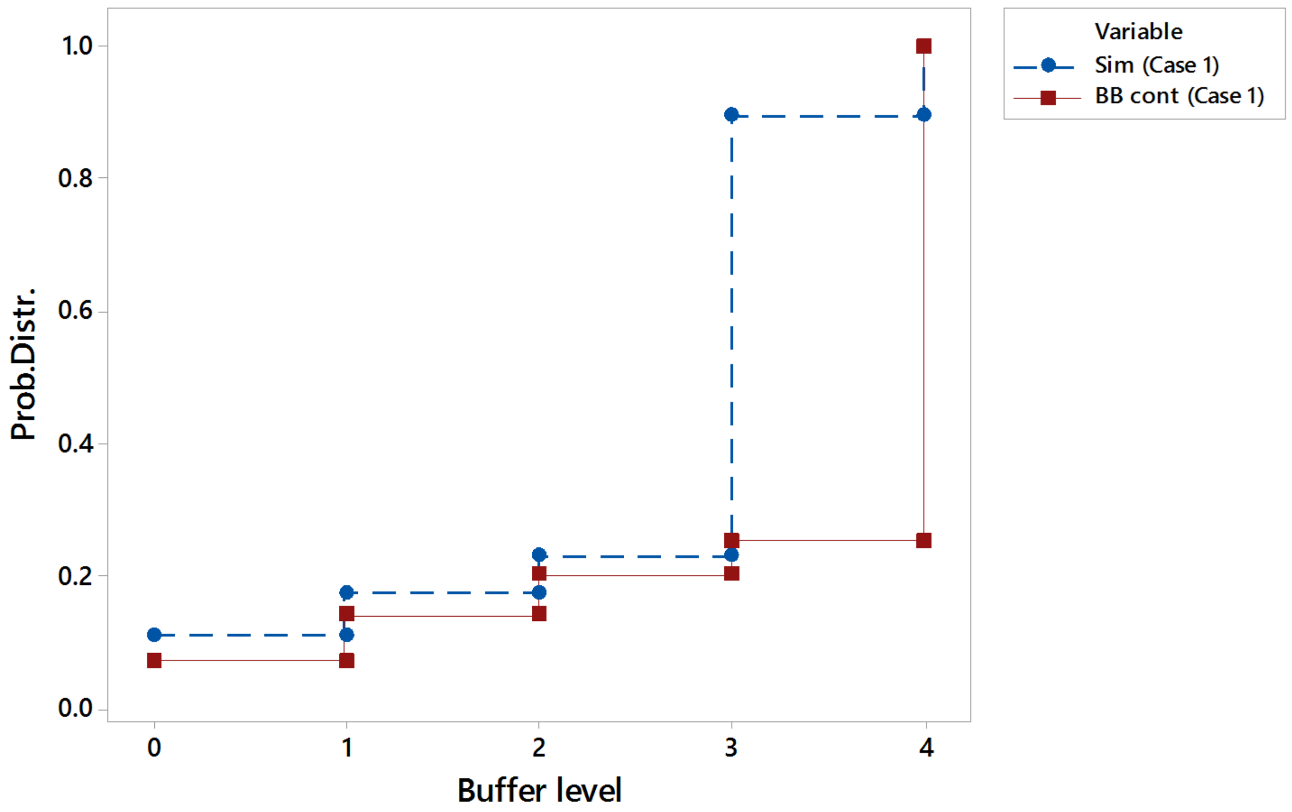


- $f\left(x, S^u S^d\right)$: probability density of joint machine state $S^u S^d$ on buffer level x
- $\pi(\Theta)$: probability mass of boundary states Θ

2. Single up – single down continuous model



Parameters							
Case i	p^u	r^u	p^d	r^d	μ^u	μ^d	N
Case 1	0.01	0.1	0.01	0.1	3	2.9	3



2. continuous model

- Literature review:
  - ***Single up and single failure***

    Gershwin and Schick (1980) – Continuous time
  - ***Single up and multiple failures***

    Levantesi, Matta and Tolio (1999) – Continuous time

    Ratti and Tolio (2013) – continuous time
  - ***Single up and Coxian repair time***:

    Ozdogru and Altiok (2003).
  - ***General Markovian machines***:

    Tan and Gershwin (2011) – continuous time
  - ***Multiple up states***:

    Tolio (2011) – continuous time
  - ***Threshold-based policies***:

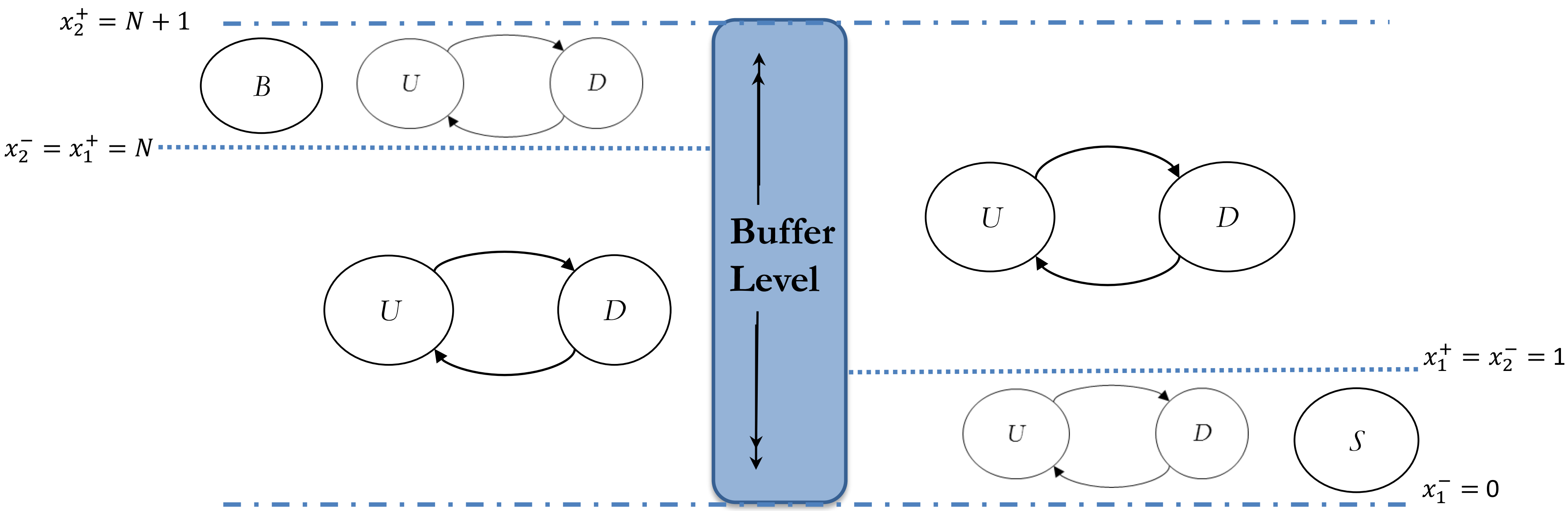
    Ratti and Tolio (2013) – continuous time

3. Proposed approach

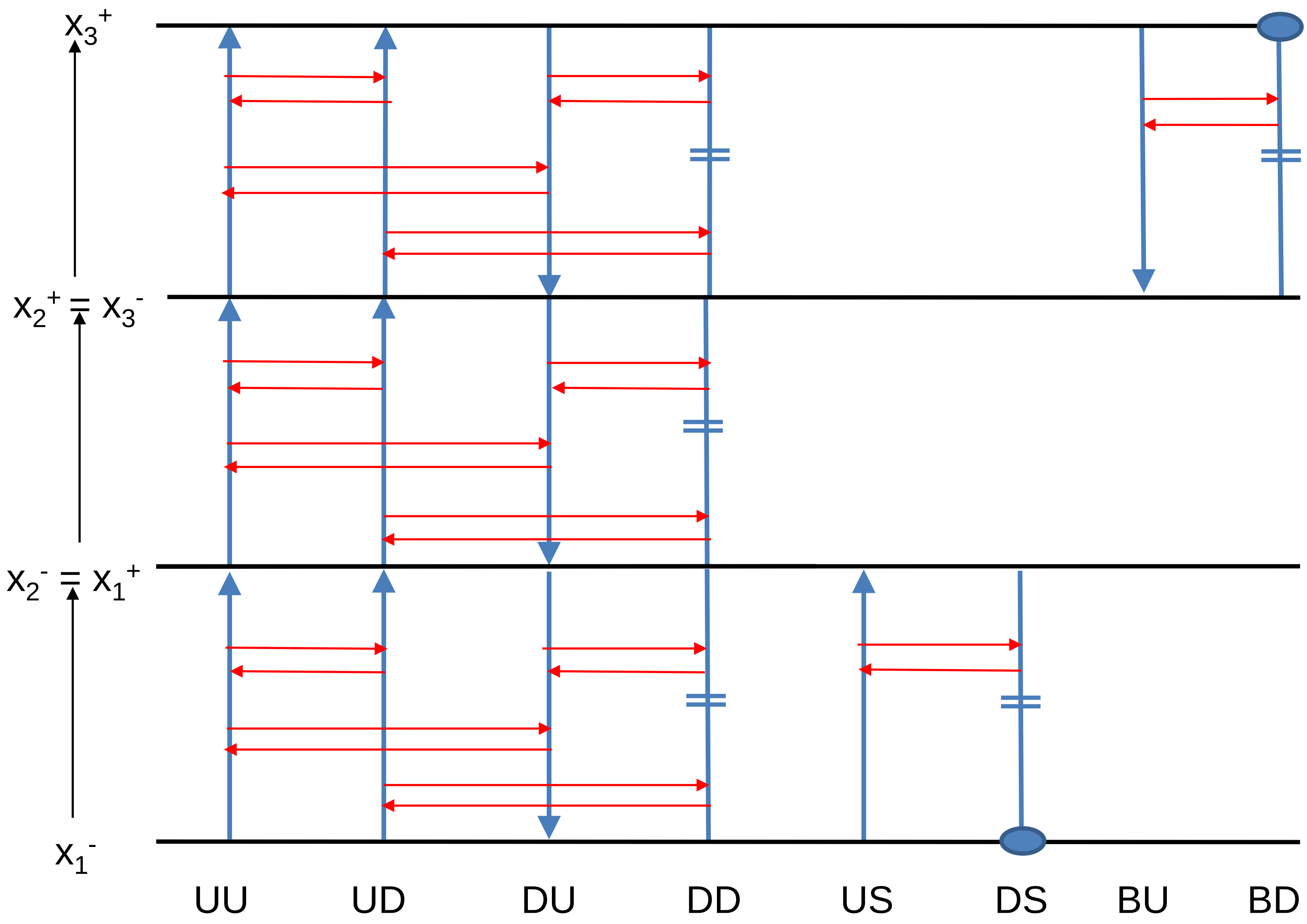
The control policy proposed acts as follows:

- It adds a blocking state (*B*) to the upstream machine when the buffer level x is in $N \leq x \leq N + 1$;
- It adds a starvation state (*S*) to the downstream machine when the buffer level x is in $0 \leq x < 1$.

➢ **Markov chain of M^u** ➢ **Markov chain of M^d**



3. Proposed approach



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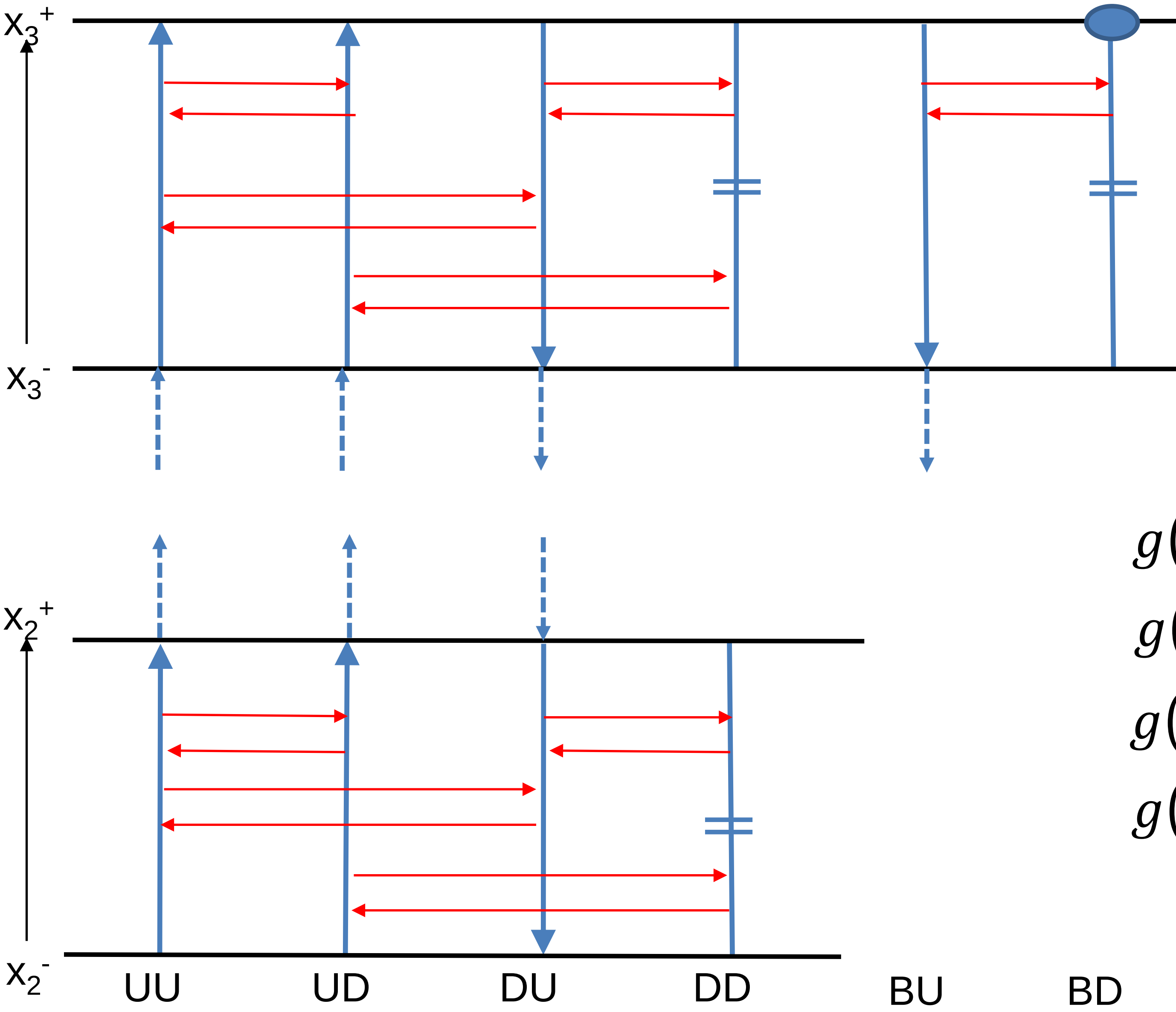
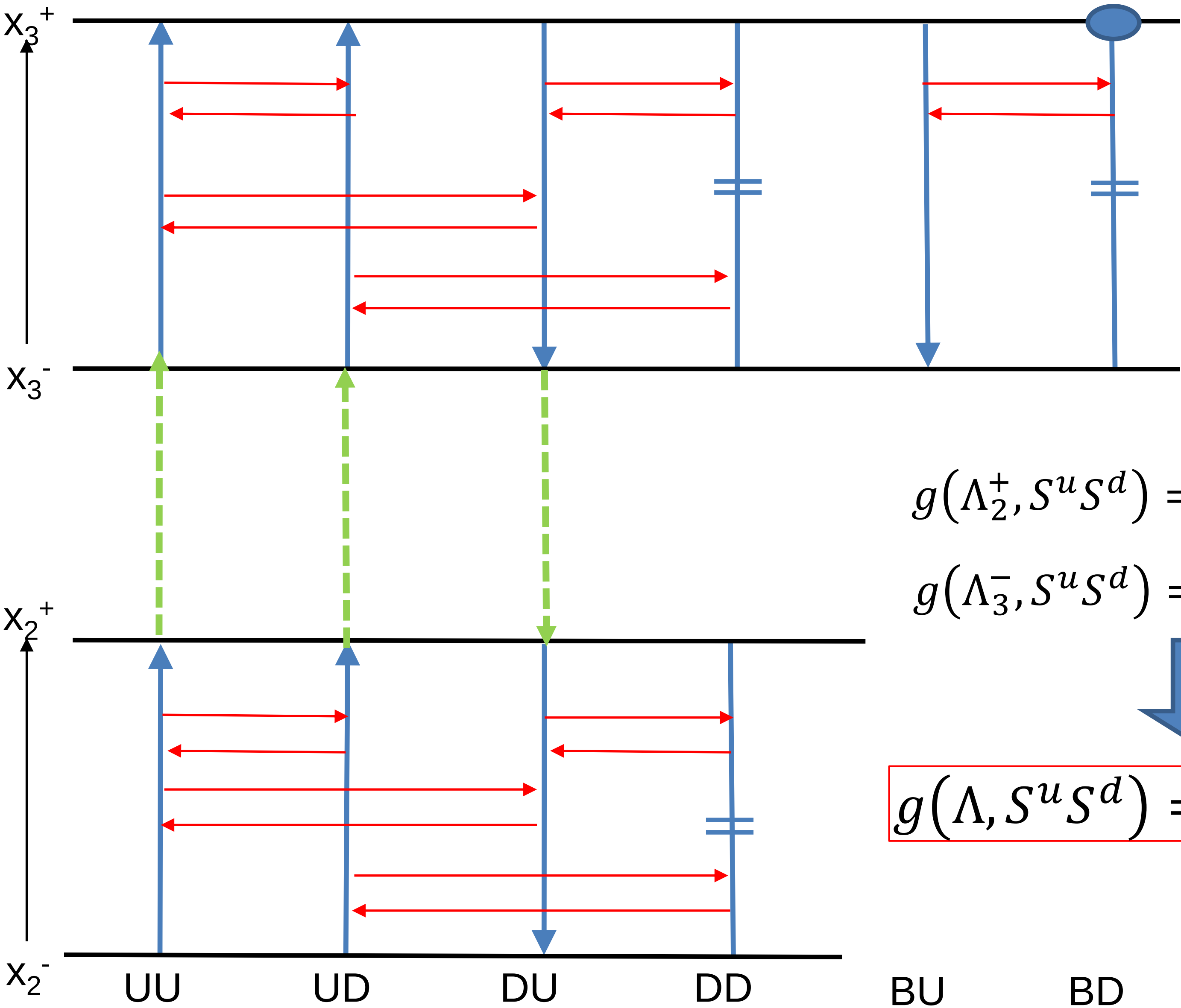


$$g\left(\Lambda_2^+, S^u S^d\right)$$
$$g\left(\Lambda_3^-, S^u S^d\right)$$
$$g\left(\Omega_2^+, S^u S^d\right)$$
$$g\left(\Omega_3^-, S^u S^d\right)$$

3. Proposed approach



$$g\left(\Lambda_2^+, S^u S^d\right) = \boldsymbol{B2_2^+} \cdot g\left(\Omega_2^+, S^u S^d\right)$$

$$g\left(\Lambda_3^-, S^u S^d\right) = \boldsymbol{B2_3^-} \cdot g\left(\Omega_3^-, S^u S^d\right)$$

$$g\left(\Lambda, S^u S^d\right) = \boldsymbol{B2} \cdot g\left(\Omega, S^u S^d\right)$$

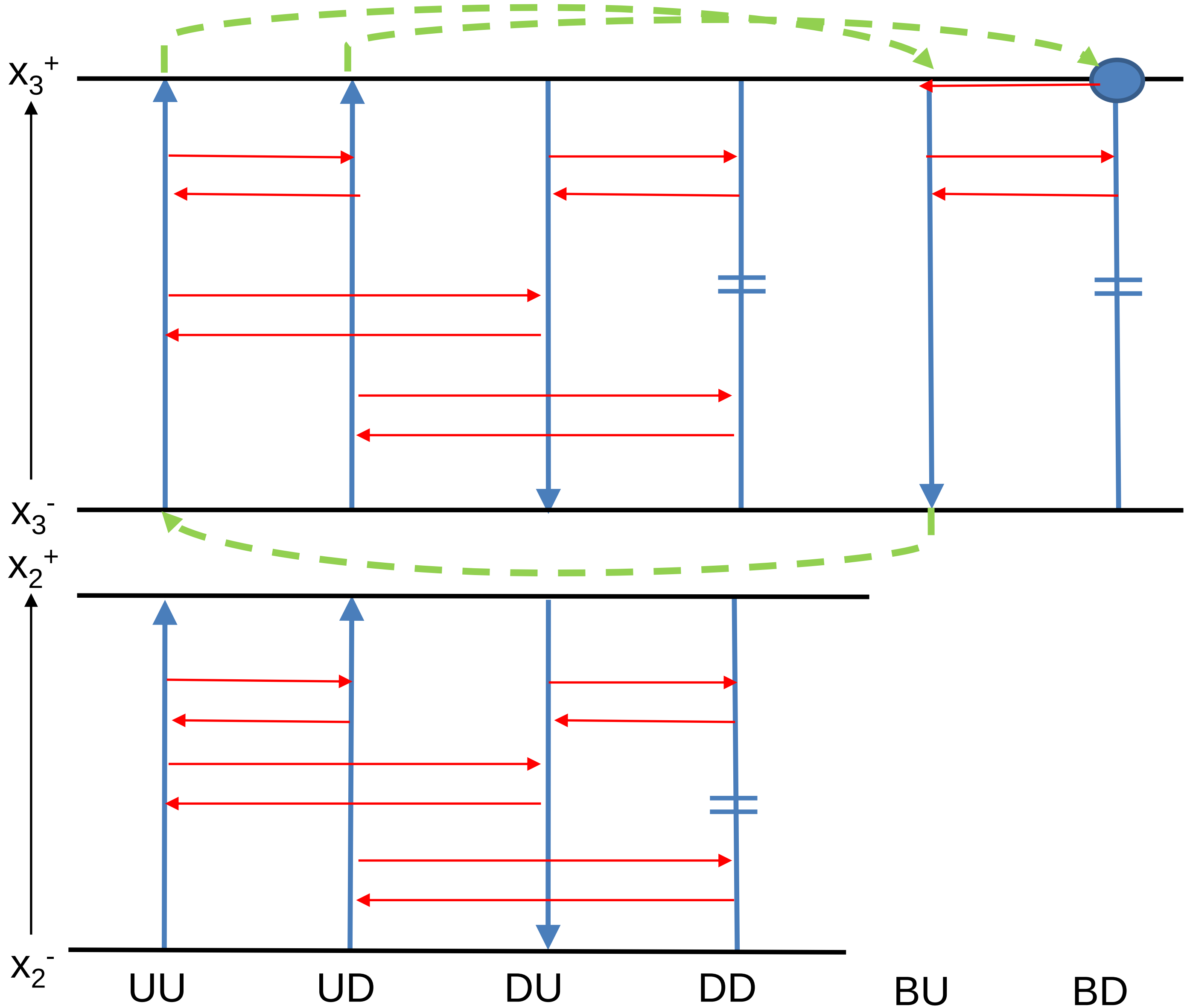
3. Proposed approach

Blocking cycle:

1. Buffer level increasing
2. M^u goes blocked
3. M^d gets repaired
4. M^u waiting for the first free place

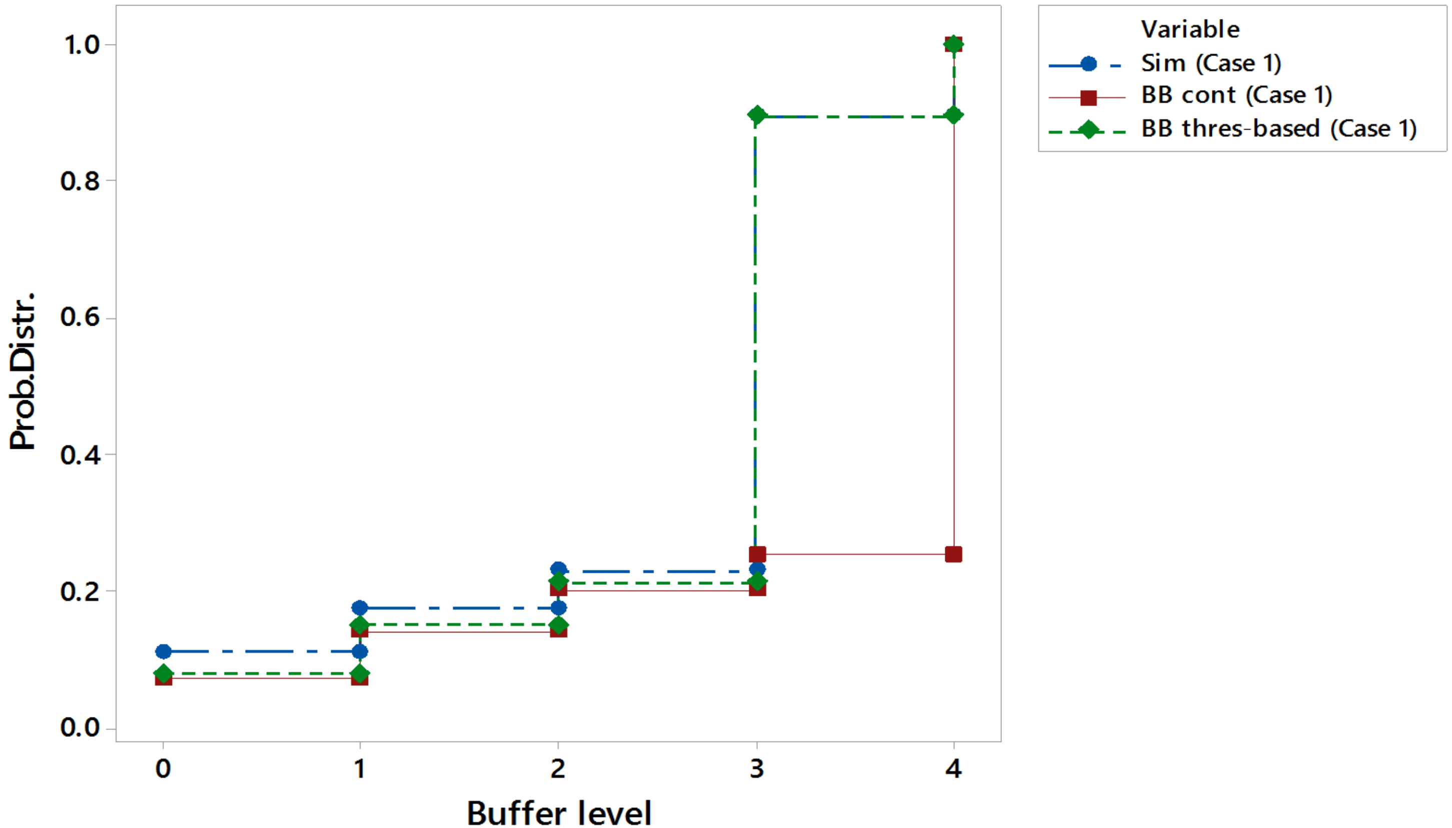
Blocking-operational cycle:

1. Buffer level increasing
2. M^u goes blocked
3. M^d empties one place in the buffer
4. M^u starts producing again.



4. Numerical results

Indeed, the proposed model closely matches the results obtained with simulation whereas, as already noticed, the continuous model has significant departures when the buffer level is close to the capacity.

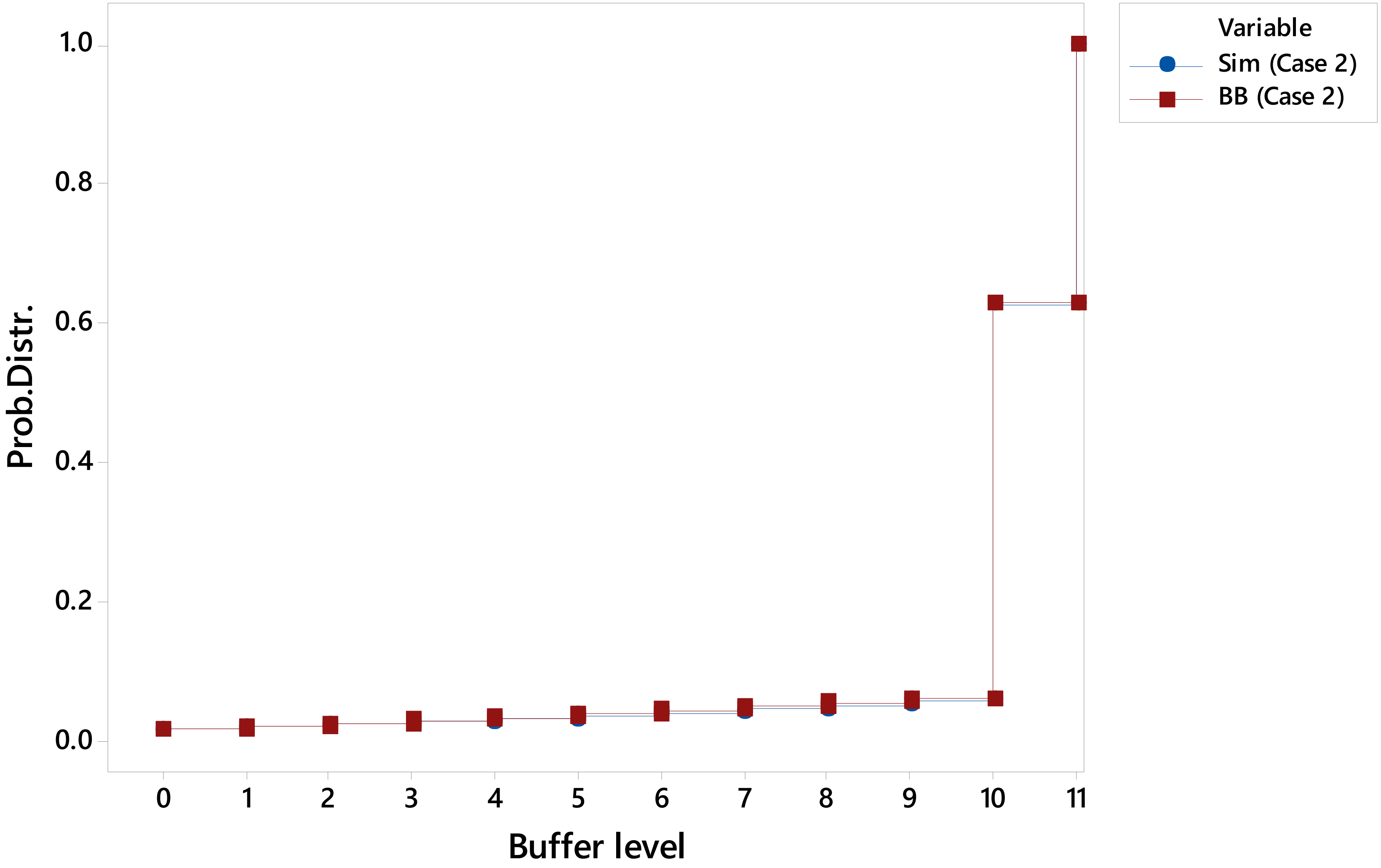


4. Numerical results

Parameters							
Case i	p^u	r^u	p^d	r^d	μ^u	μ^d	N
Case 1	0.01	0.1	0.01	0.1	3	2.9	3
Case 2	0.008	0.15	0.01	0.1	3	2	10
Case 3	0.012	0.15	0.01	0.1	3	2.9	10
Case 4	0.018	0.15	0.01	0.1	2	3	10

	Throughput			Av.Inventory		
Case i	Sim	BB_{cont}	BB_{thres}	Sim	BB_{cont}	BB_{thres}
Case 1	2.443	2.446	2.440	2.591	3.334	2.666
err%	-	0.13	0.15	-	-24.74	-2.48
Case 2	1.792	1.793	1.791	10.028	10.599	10.011
err%	-	-0.06	0.06	-	-5.71	0.17
Case 3	2.517	2.519	2.514	7.226	7.792	7.407
err%	-	- 0.06	0.13	-	-5.66	-1.81
Case 4	1.725	1.727	1.724	0.666	0.710	0.671
err%	-	0.13	0.05	-	-0.44	-0.05

4. Numerical results



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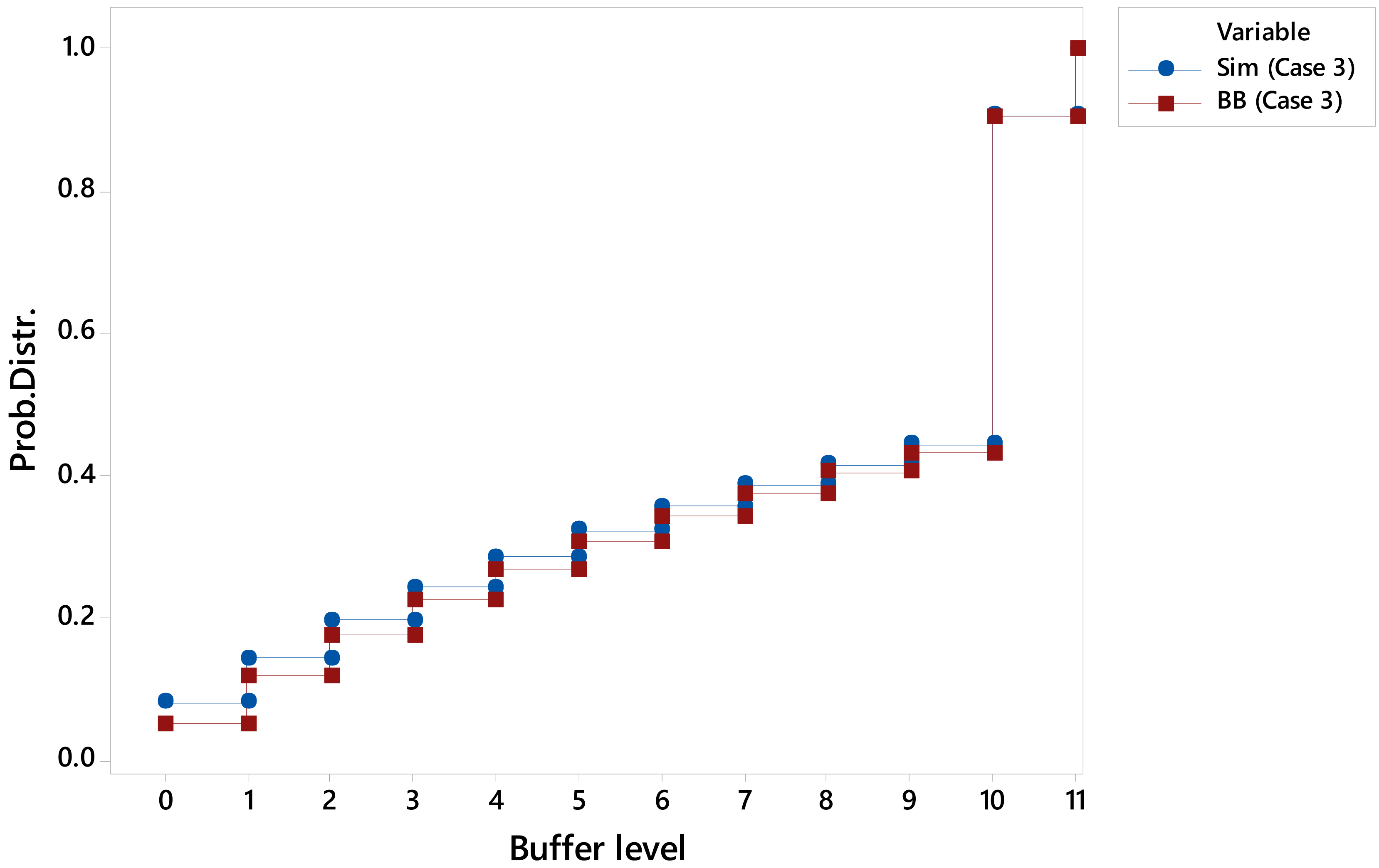

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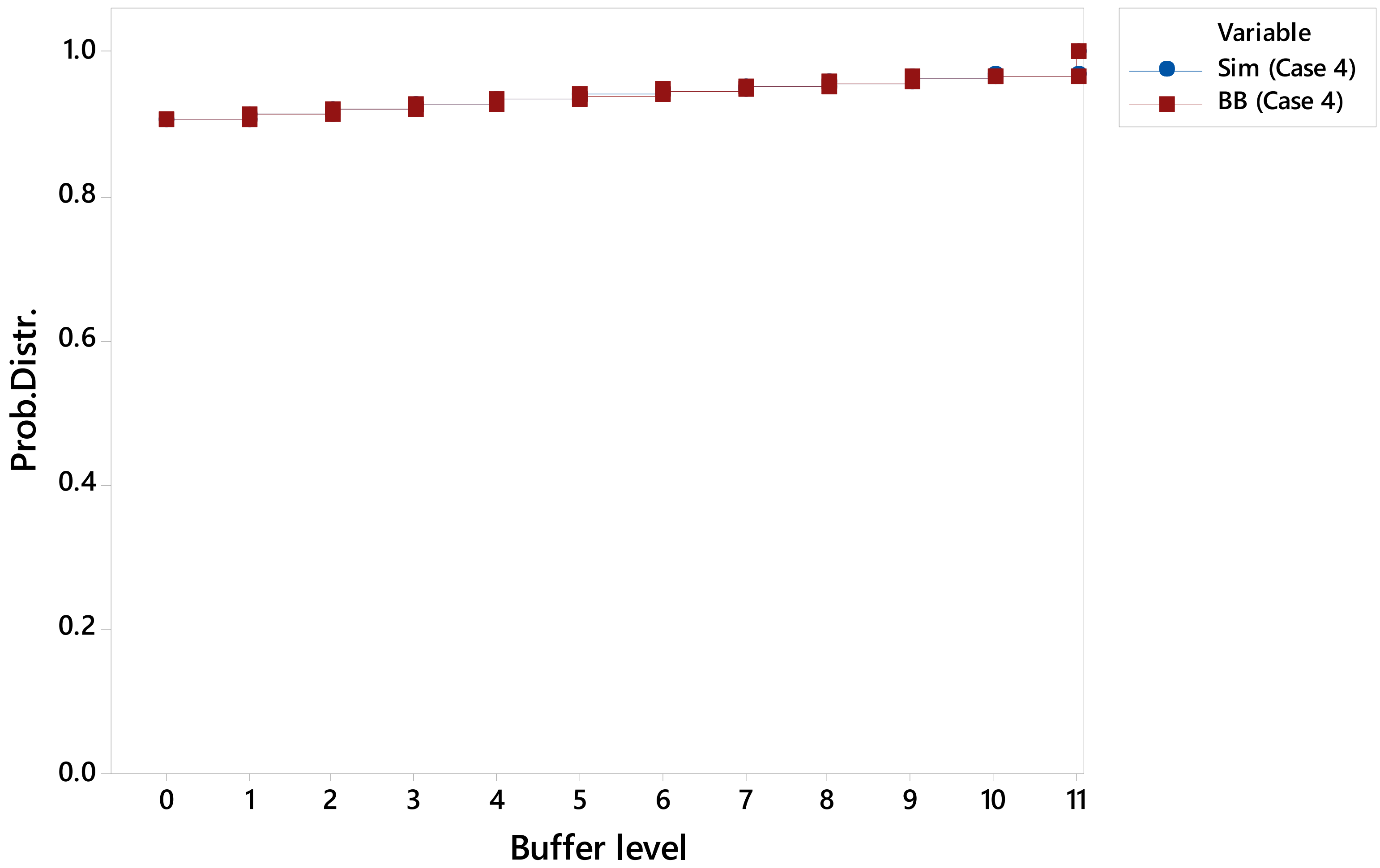

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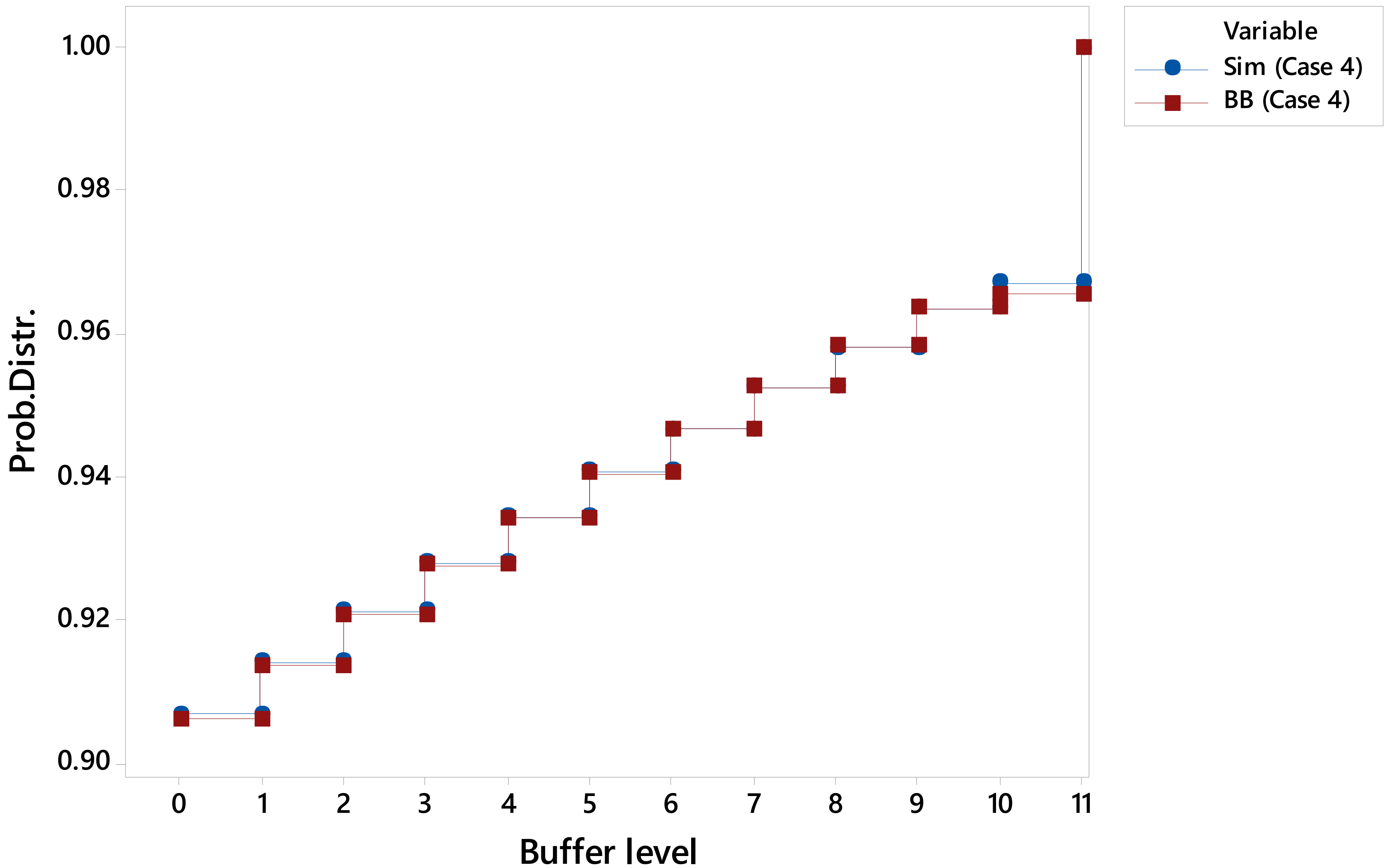

5. Conclusions and future research

The proposed method seems to be a viable approach to use continuous models in estimating the performance of discrete asynchronous two machine lines with deterministic processing times and finite buffer capacity.

- **Next steps**:
  - use the proposed two-machine line evaluation as building block in decomposition technique;
  - performance evaluation with production and transportation batches.

THANK YOU

…questions?